**Unit-1 Additional Problems**

**Rule of Sum and Product**

**Question1** – Pizza Hut is currently serving 9 kinds of individual meals- Pizzas and -9 kinds of individual Pastas. Bruce wants to try a different meal each day. How long does it take for him to try each meal once?

Ans: 9+9=18 days

**Question2** - A boy lives at X and wants to go to School at Z. From his home X he has to first reach Y and then Y to Z. He may go X to Y by either 3 bus routes or 2 train routes. From there, he can either choose 4 bus routes or 5 train routes to reach Z. How many ways are there to go from X to Z?

**Solution** − From X to Y, he can go in 3+2=5 ways (Rule of Sum). Thereafter, he can go Y to Z in 4+5=9 ways (Rule of Sum). Hence from X to Z he can go in 5×9=45 ways (Rule of Product).

**Question3:** There are 5 Chinese books, 7 English books, 10 French books How many ways to choose two books of different languages from them ?

Answer : 5 × 7 + 5 × 10 + 7 × 10 = 155 ways

**Question4:** How many bit strings of length seven are there?

Solution: Since each bit is either 0 or 1, applying the product rule, the answer is 27 = 128

**Question5:** How many different car license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

**Ans:** 26 x 26 x 26 x 10 x 10 x 10 = 17,576,000

**Question6:** Suppose variable names in a programming language can be either a single uppercase letter or an uppercase letter followed by a digit. Find the number of possible variable names.

Solution: Use the sum and product rules: 26 + 26 · 10 = 286.

**Question7:** Each user on a computer system has a password which must be six to eight characters long. Each character is an uppercase letter or digit. Each password must contain at least one digit. How many possible passwords are there?

Solution: Let P be the total number of passwords, and let P6, P7, P8 be the number of passwords of lengths 6, 7, and 8, respectively. By the sum rule P = P6 + P7 + P8. P6 = 366 − 266 (all 6 chars may be either letter or digit – all 6 chars with only letters= all 6 chars with at least one digit)

P6= 1d 5 letters+ 2d 4 letters + 3d 3letters + 4d 2 letters + 5d 1 letter + 6d= with all letters and digits - with only digits= 366 − 266

P7 = 367 − 267

P8 = 368 − 268 .

So, P = P6 + P7 + P8 = Σ i=6 to 8 (36i − 26i )

**Question8:** New company with 12 offices and 2 employees Kate and Jack

How many ways to assign different offices to Kate and Jack?

Soln: 12 x 11 ways

**Question9:** Suppose either a CS faculty or CS student must be chosen as representative for a committee. In how many ways it can be done?

There are 14 faculty, and 50 students

How many ways are there to choose the representative?

Ans: By the sum rule, 50 + 14 = 64 ways

**Question10:** Suppose one CS faculty and one CS student must be chosen as representative for a committee. There are 14 faculty, and 50 students. How many ways are there to choose the committee?

Ans: By the product rule, 50 x 14 = 700 ways

**Question11:** A student can choose a senior project from one of three lists. First list contains 23 projects; second list has 15 projects, and third has 19 projects. Also, no project appears on more than one list. How many different projects can student choose?

Ans: 23+15+19 = 57 ways

**Question11-1:** A student can choose 2 non overlapping projects from one of three lists. First list contains 23 projects; second list has 15 projects, and third has 19 projects . Also, no project appears on more than one list. How many different projects can student choose?

Ans: 23x15+15x19+19x23 = **1,57,757**

**Permutations**

**Question1:** How many permutations of the letters (ABC)DEFGH contain the string ABC as a (consecutive) substring?

Solution: We solve this by noting that this number is the same as the number of permutations of the following six objects: ABC, D, E, F, G, and H. So the answer is: 6! = 720.

**Question2:** How many different strings can be made by reordering the letters of the word “SUCCESS”?

**Solution: 7!/(2! 3!) = 420**

**Question3:** How many pairs of dance partners can be selected from a group of 12 women and 20 men?

Ans: P(20, 12) =

**Question4:** Find the number of permutations of letter of word ‘CLIMATE’

1. No restrictions
2. vowels occur in odd places.

Ans:

a) 7!

b)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **1** | 2 | **3** | 4 | **5** | 6 | **7** |

3 vowels- I,A,E and 4 odd positions-1,3,5,7 So, vowels can be arranged in 4P3 ways = 24

4 consonants – C,L,M,T 4 consonants can be arranged in 4P4 ways = 24ways

Therefore required ans is 24 X 24 = 576 ways

**Question5:** In how many ways letters of ‘MATHEMATICS’ be arranged

1. No restrictions
2. All vowels come together 4! 8!/(2!2!2!)

Ans:

a) 11!/(2! 2! 2!) ways (2A, 2M, 2T)

b) (4vowels) + 7 consonants = 8 letters

In the word 'MATHEMATICS', we'll consider all the vowels AEAI together as one letter.  
Thus, we have MTHMTCS (AEAI).  
Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice  
 Number of ways of arranging these letters =8! / ((2!)(2!))= 10080.  
  
Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.  
Number of ways of arranging these letters =4! / 2!= 12.  
  
 Required number of words = (10080 x 12) = **120960**

**Question6 :** The number of permutations of letters in ‘COMPUTER’ is 8!=

How many 5 letter word is possible from ‘COMPUTER’ = 8P5 = 6720

**Question7:** In how many ways can the English letters be arranged so that there are exactly ten letters between a and z?

Ans: The number of ways of arranging 10 letters between a and z is P(24, 10). Since we can choose either a or z to come first, there are 2P(24, 10) arrangements of this 12-letter block. For the remaining 14 letters, there are 14! arrangements. In all, there are 2P(24, 10) · 14!

**Question10:** How many permutations of the letters a, b, c, d, e, f, g contain neither the pattern bge nor eaf?

Ans: The number of total permutations is P(7, 7) = 7!. If we fix the pattern bge, then we can consider it as a single block. Thus, the number of permutations with this pattern is P(5, 5) = 5!.

Fixing the pattern eaf we have the same number, 5!. Thus we have 7! − 2(5!)

Is this correct? No.

We have taken away too many permutations: ones containing both eaf and bge. Here there are two cases, when eaf comes first and when bge comes first.

eaf cannot come before bge, so this is not a problem.

If bge comes first, it must be the case that we have bgeaf as a single block and so we have 3 blocks or 3! arrangements. Altogether we have 7! − 2(5!) + 3! = 4806

**Question11:** How many paths are possible from (1,2) to (6,9) considering one right or one up move?

Ans: there are (6-1) right moves and (9-2) up moves which are similar. So, it is like arranging 5 R and 7 U= 12! /(5! 7!) ways

**Combinations**

**Question1.** In a sequence of 10 coin tosses, how many ways can 3 heads (and 7 tails) come up?

Ans: The number of ways of choosing 3 heads out of 10 coin tosses is 10C3= 10!/(3! 7!) = 120

**Question2.** How many possible committees of five people can be chosen from 20 men and 12 women if

1. if exactly three men must be on each committee?

2. if at least four women must be on each committee?

Ans: For (1), we must choose 3 men from 20 then two women from 12. These are not mutually exclusive, thus the product rule applies. 20C3 x 12C2 = 1140 x 66 = **75240**

For (2), we consider two cases; the case where four women are chosen and the case where five women are chosen. These two cases are mutually exclusive so we use the addition rule.

For the first case we have 20C1 x 12C4 = 20 x 495

And for the second we have

20C0 X 12C5 Together we have20C1 x 12C4 + 20C0 X 12C5 = 20 x 495 +792 = **10, 692**

**Question3.** There are 28 juniors and 25 seniors. Find the number of ways of selecting

1. 9 from juniors and seniors - Ans
2. 2 juniors and 1 senior are best. A group of 9 which includes this best juniors and seniors. Ans:
3. 4 juniors and 5 seniors – Ans:

**Question4.** There are 36 girls among which 4 teams of 9 girls to be formed. In how many ways it could be done?

Ans: = 2.145x 1019 ways.

**Question5.** There are 12 magazines. In how many ways 5 magazines be selected?

Ans: = 792 ways

**Question6.** A committee of 12 is to be selected from 10 men and 10 women. In how many ways selection can be done if

1. No restrictions – Ans:
2. There must be 6 men and 6 women – Ans:
3. There must be even number of women –

Ans: women count may be 0,2,4,6,8 or 10

++++ =

1. There must be more women than men

Ans: M W

+

1. There must be atleast 8 men

Ans: M W

**Question7.** How many bytes contain

1. **Exactly two 1’s –** Ans = 28
2. **Exactly 4 1’s -** Ans = 70
3. **Atleast 6 1’s -** Ans
4. **Exactly 6 1’s -** Ans

**Question8.** There are 12 players. In how many ways 5 players be selected if

1. No restrictions - Ans = 792 ways
2. Team which include weakest and strongest players – Ans: = 120 ways

**Combinations with repetitions**

**Question1.** In how many ways we can distribute 10 identical marbles among 6 distinct containers?

i ) with no restrictions

Ans: Here n=6, r=10 = ===== =

ii) with no container left empty?

Ans: Here n=6, r=10-6=4 = =

**Question2.** Find the number of integer solutions of x1+x2+x3+x4+x5 = 30 with x1≥2, x2 ≥3, x3 ≥4, x4 ≥2, x5 ≥0

Ans: Here n=5, r=30-2-3-4-2=19 =

Ans: 8855

**Question3.** Find the number of non negative integer solutions

of x1+x2+x3+x4+x5 = 8

Ans: here n=5 r=8 so ans is